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Flow and heat transfer in round vertical buoyant jets

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Abstract—The application of the algebraic stress modeling method (ASM) for calculating the flow in a turbulent vertical round jet is described. The ASM method yields algebraic equations for Reynolds stresses $\langle u_i'u_j' \rangle$ and turbulent heat flux components $\langle u_i'T' \rangle$. Transport equations are solved for the turbulent kinetic energy k, its dissipation rate ε and mean square of temperature fluctuations $\langle T'^2 \rangle$. A new version of the model has been used allowing one to account for the effect of normal Reynolds stresses on energy redistribution in the spectrum of turbulent fluctuations. To model these effects, an additional term has been included into the equation for ε. A new effective approach to solving the governing system of partial differential equations is suggested based on the introduction of mathematical variables. Examples of numerical calculations are presented which are compared with the available experimental data of other authors. It has been found that they agree satisfactorily with the results of measurements.

1. INTRODUCTION

IN RECENT years, the problem of quantitative description of turbulent jet transfer of momentum, heat and substance in the field of mass forces has claimed the attention of many scientists and engineers. First of all, the interest in this problem is stimulated by the possibility of obtaining some insight into the nature of the phenomenon and its trends by varying initial and boundary conditions and to exert a marked influence on the process. By now, a great deal of experimental information has been accumulated and correlated [1–12] on submerged jets, plumes and buoyant jet flows. This knowledge has turned out to be sufficient to develop and suggest useful mathematical models of the given phenomenon. The differential–parametric models and the idea of 'differential–algebraic' modeling that combine the relative simplicity and universality with modern computerization have turned out to be most popular. The results of such modeling have been discussed in refs. [13–19].

At the same time, the results of applying these models to solve the problem of the development of an axisymmetric submerged jet have proved to be far from reality. This justified the use of other variants of closing model equations, but still within the confines of the same approach [20–22].

It is the augmentation of the equation for dissipation rate ε with a new term, which can be interpreted as an additional contribution to production due to vortex stretching under the effect of averaged flow (for the plane flow geometry this term is equal to zero, since the length of averaged vortex lines remains unknown) [20]. According to ref. [21], energy redistribution between different regions of the spectrum of turbulent fluctuations is mainly caused by translational strains (or by normal Reynolds stresses). Taking this fact into consideration, the authors of ref. [21] have also modified the equation for ε. In ref. [22] another version of the (k–ε) turbulelnt model has been suggested which combines the properties of the models proposed in refs. [20, 21].

The latter points to the fact that an adequate modeling of turbulent jet flows is still fairly uncertain and warrants further research.

Besides, recently there has been a compelling need for developing numerical procedures permitting the prediction of mixing processes in jet flows with high efficiency and accuracy. Such methods should combine stability with acceptable time expenditures, since along with integration of continuity, momentum and energy equations one has to solve a large number of equations describing the transfer of turbulent characteristics.

2. MATHEMATICAL MODEL

Let us consider a round turbulent buoyant jet, which on the nozzle cut has the radius r_0, initial velocity u_0, temperature T_0 and fluid density $\rho_0 (\rho_0 < \rho_\infty)$ and which propagates vertically upward in an isothermal medium under the effect of the Archimedes forces and initial momentum.

The initial system of differential equations for calculating the steady-state turbulent flow in the jet is of the following form

$$\frac{\partial}{\partial x}(yu) + \frac{\partial}{\partial y}(yv) = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{y}\frac{\partial}{\partial y}(-y\langle u'v' \rangle) + g\beta\Delta T;$$

$$u\frac{\partial k}{\partial x} + v\frac{\partial k}{\partial y} = \frac{1}{y}\frac{\partial}{\partial y}\left(c_k y \frac{k}{\varepsilon}\langle v'^2 \rangle \frac{\partial k}{\partial y}\right) + P + G - \varepsilon;$$

NOMENCLATURE

d_0	jet diameter at the outlet
F	Froude number, $u_0^2/g\beta\Delta T_0 r_0$
g	free fall acceleration
k	turbulent kinetic energy
P	production of turbulent kinetic energy
R	ratio of temperature and velocity fluctuations time scales
T	mean temperature, $\Delta T = T - T_\infty$
T'	temperature fluctuation
$\langle T'^2 \rangle$	mean square temperature fluctuation
u, v	averaged velocity components
u', v', w'	fluctuating velocity components
$\langle u'v' \rangle$	turbulent shear stress
$\langle u'^2 \rangle$, $\langle v'^2 \rangle$, $\langle w'^2 \rangle$	normal Reynolds stresses
x, y	coordinates
$y_{0.5u}$, $y_{0.5t}$	jet half-width.

Greek symbols

β	coefficient of volumetric expansion
ε	dissipation rate k
ε_t	dissipation rate of the quantity $\langle T'^2 \rangle$
ν_t	turbulent viscosity coefficient
ρ	fluid density
σ_t	turbulent Prandtl number.

Subscripts

c	jet axial line
m	maximum value
0	exit cross-section of the jet
1	non-dimensional value
∞	ambient fluid.

$$u\frac{\partial \varepsilon}{\partial x} + v\frac{\partial \varepsilon}{\partial y} = \frac{1}{y}\frac{\partial}{\partial y}\left(c_\varepsilon y \frac{k}{\varepsilon}\langle v'^2\rangle \frac{\partial \varepsilon}{\partial y}\right) + c_{\varepsilon 1}\frac{\varepsilon}{k}(P+G) - c_{\varepsilon 2}\frac{\varepsilon^2}{k};$$

$$u\frac{\partial \langle T'^2\rangle}{\partial x} + v\frac{\partial \langle T'^2\rangle}{\partial y} = \frac{1}{y}\frac{\partial}{\partial y}\left(c_T y \frac{k^2}{\varepsilon}\frac{\partial \langle T'^2\rangle}{\partial y}\right) - 2\langle v'T'\rangle \frac{\partial T}{\partial y} - c_{T1}\frac{\varepsilon}{k}\langle T'^2\rangle;$$

$$G = g\beta\langle u'T'\rangle. \qquad (1)$$

Here, the term describing the rate of turbulent production in the equation of turbulent energy balance is defined by the expression

$$P = -\langle u'v'\rangle \frac{\partial u}{\partial y} - (\langle u'^2\rangle - \langle v'^2\rangle)\frac{\partial u}{\partial x}. \qquad (2)$$

Note that the second term of equality (2) which comprises the rate of irrotational strain, is usually neglected.

Next, the existence of the following algebraic relations for Reynolds stresses and turbulent heat flux components is postulated

$$-\langle u'v'\rangle = \frac{1-c_0}{c_1}\frac{\langle v'^2\rangle}{k}\left[1 + \frac{g\beta k}{c_n \varepsilon}\frac{\frac{\partial T}{\partial y}}{\frac{\partial u}{\partial y}}\right]\frac{k^2}{\varepsilon}\frac{\partial u}{\partial y},$$

$$\langle v'^2\rangle = c_2 k, \quad -\langle v'T'\rangle = \frac{1}{c_h}\frac{\langle v'^2\rangle}{k}\frac{k^2}{\varepsilon}\frac{\partial T}{\partial t},$$

$$\langle u'T'\rangle = \frac{1}{c_n}\frac{k}{\varepsilon}\left[-\langle u'v'\rangle \frac{\partial T}{\partial y} - (1-c_{h1})\langle v'T'\rangle \frac{\partial u}{\partial y} + g\beta(1-c_{h1})\langle T'^2\rangle\right]. \qquad (3)$$

Equations (1)–(3) form a completely closed system complying with the so-called algebraic model of Reynolds stresses, which includes 11 empirical constants such as [15, 16]:

$$c_0 = 0.55, \quad c_1 = 2.2, \quad c_2 = 0.53, \quad c_\varepsilon = 0.15,$$
$$c_k = 0.225, \quad c_{\varepsilon 1} = 1.43, \quad c_{\varepsilon 2} = 1.92, \quad c_T = 0.13,$$
$$c_{T1} = 1.25, \quad c_h = 3.2, \quad c_{h1} = 0.5. \qquad (4)$$

However, in contrast to refs. [13, 15–19], in the present work a new version of the ASM has been employed in which the contribution of irrotational strain in production P remains unaffected, but in the ε-equation, this term is multiplied by the factor $c_{\varepsilon 3}$, rather than $c_{\varepsilon 1}$, by which the term describing the vortical portion the strain is multiplied. According to ref. [21], this makes it possible to more accurately predict the rate of expansion of round jets than is done by the familiar $(k-\varepsilon)$ model, and to give up the empirical relations of the form:

$$c_\mu = 0.09(1-0.465H), \quad c_{\varepsilon 2} = 1.92(1-0.035H),$$

$$H = \left|\frac{y_{0.5u}}{u_c}\frac{du_c}{dx}\right|^{1/5}, \qquad (5)$$

which are usually employed in standard versions of the $(k-\varepsilon)$ and ASM-models when calculating round jets.

In accordance with the problem statement, the system of the foregoing equations should be solved under the following initial boundary-value conditions:

$x = 0$

$$\begin{cases} u = u_0, \quad T = T_0, \quad k = k_0, \quad \varepsilon = \varepsilon_0, \\ \quad \langle T'^2\rangle = \langle T'^2\rangle_0 \qquad \text{for} \quad 0 \leqslant y < r_0; \\ u = 0, \quad T = T_\infty, \quad k = 0, \quad \varepsilon = 0, \\ \quad \langle T'^2\rangle = 0 \qquad \text{for} \quad r_0 \leqslant y < \infty; \end{cases}$$

$$x>0\begin{cases} y=0: & v=\dfrac{\partial u}{\partial y}=\dfrac{\partial T}{\partial y}=\dfrac{\partial k}{\partial y} \\ & =\dfrac{\partial \varepsilon}{\partial y}=\dfrac{\partial\langle T'^2\rangle}{\partial y}=0; \\ y\to\infty: & u,k,\varepsilon,\quad \langle T'^2\rangle\to 0,\quad T\to T_\infty. \end{cases} \tag{6}$$

3. METHOD OF SOLUTION

Integration of equations (1)–(6) after the transition to dimensionless variables

$$U=\frac{u}{u_0},\quad V=\frac{v}{u_0},\quad X=\frac{x}{r_0},\quad Y=\frac{y}{r_0},$$
$$K=\frac{k}{u_0^2},\quad E=\frac{\varepsilon r_0}{u_0^3},\quad \theta=\frac{\Delta T}{\Delta T_0},\quad q=\frac{\langle T'^2\rangle}{\Delta T_0^2} \tag{7}$$

with the given initial distributions for u_0, T_0, k_0, ε_0, $\langle T'^2\rangle_0$, integration of equations (1)–(6) was earlier performed with the help of a number of various numerical schemes. Usually, in the course of solution, the flow region is covered with a rectangular grid on the x, y plane with the Δx, Δy step. Next, finite-difference equations, approximating initial differential ones are solved by the method of iterations by means of fitting. Such analysis schemes of problem (1)–(6) have a common drawback in that they require a lot of computer time, because abandonment of iteration when solving each of the finite-difference equations can lead to a situation when the general iteration process can turn out to be nonconvergent.

In the present work a new approach has been realized. It consists of the introduction of the following mathematical variables

$$X=X,\quad \eta=\left(2\int_0^Y U\theta Y\,\mathrm{d}Y\right)^{1/2}, \tag{8}$$

in which the system of equations (1)–(3) acquires the form

$$\frac{\partial U}{\partial X}=2c_1^*\frac{K^2}{E}\frac{\theta}{\eta}\frac{\partial U}{\partial \eta}+c_1^*\frac{Y^2}{\eta^2}\frac{K^2}{E}\theta^2$$
$$\times\left[U\frac{\partial^2 U}{\partial\eta^2}+\left(1-\frac{c_3^*}{c_1^*}\right)\frac{U}{\theta}\frac{\partial U}{\partial\eta}\frac{\partial\theta}{\partial\eta}+\left(\frac{\partial U}{\partial\eta}\right)^2\right.$$
$$\left.-\frac{U}{\eta}\frac{\partial U}{\partial\eta}+2\frac{U}{K}\frac{\partial K}{\partial\eta}\frac{\partial U}{\partial\eta}-\frac{U}{E}\frac{\partial E}{\partial\eta}\frac{\partial U}{\partial\eta}\right]$$
$$+\frac{2}{F}c_2^*\frac{K^3}{E^2}\frac{\theta}{\eta}\frac{\partial\theta}{\partial\eta}+\frac{1}{F}c_2^*\frac{Y^2}{\eta^2}\frac{K^3}{E^2}\theta^2$$
$$\times\left[U\frac{\partial^2\theta}{\partial\eta^2}+\frac{U}{\theta}\left(\frac{\partial\theta}{\partial\eta}\right)^2+\frac{\partial U}{\partial\eta}\frac{\partial\theta}{\partial\eta}-\frac{U}{\eta}\frac{\partial\theta}{\partial\eta}\right.$$
$$\left.+3\frac{U}{K}\frac{\partial K}{\partial\eta}\frac{\partial\theta}{\partial\eta}-2\frac{U}{E}\frac{\partial E}{\partial\eta}\frac{\partial\theta}{\partial\eta}\right]+\frac{1}{F}\frac{\theta}{U};$$

$$\frac{\partial\theta}{\partial X}=c_3^*\frac{Y^2}{\eta^2}\frac{K^2}{E}\theta^2\left[U\frac{\partial^2\theta}{\partial\eta^2}+\frac{\partial U}{\partial\eta}\frac{\partial\theta}{\partial\eta}-\frac{U}{\eta}\frac{\partial\theta}{\partial\eta}\right.$$
$$\left.+2\frac{U}{K}\frac{\partial K}{\partial\eta}\frac{\partial\theta}{\partial\eta}-\frac{U}{E}\frac{\partial E}{\partial\eta}\frac{\partial\theta}{\partial\eta}\right]+2c_3^*\frac{K^2}{E}\frac{\theta}{\eta}\frac{\partial\theta}{\partial\eta};$$

$$\frac{\partial K}{\partial X}=2c_k^*\frac{K^2}{E}\frac{\theta}{\eta}\frac{\partial K}{\partial\eta}+c_k^*\frac{Y^2}{\eta^2}\frac{K^2}{E}\theta^2$$
$$\times\left[U\frac{\partial^2 K}{\partial\eta^2}+\left(1-\frac{c_3^*}{c_k^*}\right)\frac{U}{\theta}\frac{\partial\theta}{\partial\eta}\frac{\partial K}{\partial\eta}+\frac{\partial U}{\partial\eta}\frac{\partial K}{\partial\eta}\right.$$
$$-\frac{U}{\eta}\frac{\partial K}{\partial\eta}+2\frac{U}{K}\left(\frac{\partial K}{\partial\eta}\right)^2-\frac{U}{E}\frac{\partial E}{\partial\eta}\frac{\partial K}{\partial\eta}+\frac{c_1^*}{c_k^*}U\left(\frac{\partial U}{\partial\eta}\right)^2$$
$$+\frac{1}{F}\frac{c_7^*}{c_k^*}\frac{UK}{E}\frac{\partial U}{\partial\eta}\frac{\partial\theta}{\partial\eta}-\frac{c_u c_3^*}{c_k^*}\frac{K}{\theta}\frac{\partial\theta}{\partial\eta}\frac{\partial U}{\partial\eta}$$
$$+\frac{1}{F^2}\frac{c_5^*}{c_k^*}\frac{K^2}{E^2}U\left(\frac{\partial\theta}{\partial\eta}\right)^2-c_u\frac{K}{U}\frac{\partial U}{\partial X}$$
$$+c_u\frac{Y}{\eta}\frac{KV\theta}{U}\frac{\partial U}{\partial\eta}+\frac{1}{F^2}c_6^*\frac{Kq}{EU}-\frac{E}{U};$$

$$\frac{\partial q}{\partial X}=2c_T\frac{K^2}{E}\frac{\theta}{\eta}\frac{\partial q}{\partial\eta}+c_T\frac{K^2}{E}\frac{Y^2}{\eta^2}\theta^2$$
$$\times\left[U\frac{\partial^2 q}{\partial\eta^2}+\left(1-\frac{c_3^*}{c_T}\right)\frac{U}{\theta}\frac{\partial\theta}{\partial\eta}\frac{\partial q}{\partial\eta}+\frac{\partial U}{\partial\eta}\frac{\partial q}{\partial\eta}\right.$$
$$+2\frac{U}{K}\frac{\partial K}{\partial\eta}\frac{\partial q}{\partial\eta}-\frac{U}{\eta}\frac{\partial q}{\partial\eta}-\frac{U}{E}\frac{\partial E}{\partial\eta}\frac{\partial q}{\partial\eta}$$
$$\left.+2\frac{c_3^*}{c_T}U\left(\frac{\partial\theta}{\partial\eta}\right)^2\right]-c_{T1}\frac{Eq}{KU};$$

$$\frac{\partial E}{\partial X}=2c_\varepsilon^*\frac{K^2}{E}\frac{\theta}{\eta}\frac{\partial E}{\partial\eta}+\frac{K^2}{E}\frac{Y^2}{\eta^2}\theta^2\left[c_\varepsilon^*\left(U\frac{\partial^2 E}{\partial\eta^2}\right.\right.$$
$$+\left(1-\frac{c_3^*}{c_\varepsilon^*}\right)\frac{U}{\theta}\frac{\partial\theta}{\partial\eta}\frac{\partial E}{\partial\eta}+\frac{\partial U}{\partial\eta}\frac{\partial E}{\partial\eta}+2\frac{U}{K}\frac{\partial K}{\partial\eta}\frac{\partial E}{\partial\eta}$$
$$\left.-\frac{U}{\eta}\frac{\partial E}{\partial\eta}-\frac{U}{E}\left(\frac{\partial E}{\partial\eta}\right)^2\right)+c_{\varepsilon 1}\left(c_1^*\frac{UE}{K}\left(\frac{\partial U}{\partial\eta}\right)^2\right.$$
$$\left.+\frac{1}{F}c_7^*U\frac{\partial\theta}{\partial\eta}\frac{\partial U}{\partial\eta}+\frac{1}{F^2}c_5^*\frac{KU}{E}\left(\frac{\partial\theta}{\partial\eta}\right)^2\right)$$
$$\left.-c_{\varepsilon 3}c_u c_3^*\frac{E}{\theta}\frac{\partial\theta}{\partial\eta}\frac{\partial U}{\partial\eta}\right]+\frac{1}{F^2}c_{\varepsilon 1}c_6^*\frac{q}{U}-c_{\varepsilon 3}c_u\frac{E}{U}\frac{\partial U}{\partial X}$$
$$+c_{\varepsilon 3}c_u\frac{Y}{\eta}\frac{EV\theta}{U}\frac{\partial U}{\partial\eta}-c_{\varepsilon 2}\frac{E^2}{KU}, \tag{9}$$

where the transverse averaged velocity component V and the coordinate Y are defined by the expressions

$$V = -c_1^* \frac{Y}{\eta} \frac{K^2}{E} \theta \frac{\partial U}{\partial \eta} - 2c_1^* \frac{U}{Y} \int_0^\eta \frac{Y^2 K^2 \theta}{\eta E U^2} \left(\frac{\partial U}{\partial \eta} \right)^2 d\eta$$

$$- \frac{1}{F} c_2^* \frac{Y}{\eta} \frac{K^3}{E^2} \theta \frac{\partial \theta}{\partial \eta} - \frac{2}{F} c_2^* \frac{U}{Y} \int_0^\eta \frac{Y^2 K^3 \theta}{\eta E^2 U^2} \frac{\partial U}{\partial \eta}$$

$$\times \frac{\partial \theta}{\partial \eta} d\eta - \frac{1}{F} \frac{U}{Y} \int_0^\eta \frac{\eta \, d\eta}{U^3};$$

$$Y = \left(2 \int_0^\eta \frac{\eta \, d\eta}{U\theta} \right)^{1/2}. \tag{10}$$

Here

$$c_1^* = \frac{1-c_0}{c_1} c_2, \quad c_2^* = \frac{1-c_0}{c_1} \frac{c_2}{c_h}, \quad c_3^* = \frac{c_2}{c_h},$$

$$c_5^* = \frac{1-c_0}{c_1} \frac{c_2}{c_h^2}, \quad c_6^* = \frac{1-c_{h1}}{c_h}, \quad c_k^* = c_2 c_k,$$

$$c_\varepsilon^* = c_2 c_\varepsilon, \quad c_7^* = 2 \frac{1-c_0}{c_1} \frac{c_2}{c_h} + \frac{c_2(1-c_{h1})}{c_h^2}. \tag{11}$$

The proposed method has two advantages. First, in the X, η system, the infinite interval over Y passes into the finite interval of the variance of η from 0 to I (in the case of uniform initial fields U and θ, $I = 1$) and, therefore, the integration domain transforms into band $0 \leqslant \eta \leqslant I$, $X \geqslant 0$. This partially eliminates the difficulties associated with the necessity to impose artificial boundary conditions in the restricted computational domain. Second, the employment of equation (8) ensures an automatic compliance with the condition of the initial 'heat momentum' conservation along the jet. This eliminates the necessity to control in all the cross section of the flow the satisfaction of the equality

$$Q_0 = 2\pi \int_0^\infty \rho u (T - T_\infty) y \, dy.$$

Next, the flow region is split into n bands, and on each of the straight lines $\eta = \eta_i$ ($i = 1, 2, 3, \ldots, n+1$) the derivatives over the variable η are replaced by their three-point central-difference analogs; as a result, the initial differential equations (9), (10) transform to a system of $5n$ first-order ordinary differential equations (in the calculations it was assumed that $n = 50$). By assigning initial conditions for velocity, temperature, turbulent kinetic energy, its dissipation rate and for the mean-square temperature fluctuation at the nozzle cut ($X = 0$) or at a certain section X^* (they can be those modeled or taken directly from experiment), the Cauchy problem is obtained which is integrated by the standard Runge–Kutta method with automatic selection of step along the dimensionless coordinate X. Let us assume that the problem has been solved, i.e. that the functions $U(X, \eta)$, $\theta(X, \eta)$, $K(X, \eta)$, $E(X, \eta)$, and $q(X, \eta)$ have been constructed. Then, to complete the solution, one should resort to equality (10) which provides the transition of the obtained functions into the physical plane X, Y.

4. DISCUSSION OF THE CALCULATED RESULTS

Calculations were started at the outlet nozzle section ($X = 0$), in which uniform profiles of U, θ and uniform distributions of K, E, q were assumed.

$$U_0 = 1, \quad \theta_0 = 1, \quad K_0 = 0.05,$$
$$E_0 = 0.02, \quad q_0 = 0.05 \tag{12}$$

and continued until the profiles became self-similar.

The calculations were performed in two steps: first, hydrodynamic and thermal characteristics of forced flow in a round jet were investigated, and then numerical simulation of the effect of free convection on the turbulence structure in a vertical flow was made.

4.1. *Forced round jet*

The first stage of calculations was primarily of test character, since the dynamics of the round jet development was already studied within the framework of the modified (k–ε) turbulence model (MM) [21].

Mixing processes in a round jet were analyzed not by the ASM method, but by a simpler (k–ε) turbulence model with allowance made for the contribution of irrotational strains into turbulent energy transfer in the spectrum of fluctuations. In this case, the shear stress was calculated using the hypothesis on turbulent viscosity

$$-\langle u'v' \rangle = v_t \frac{\partial u}{\partial y}, \quad v_t = c_\mu \frac{k^2}{\varepsilon},$$

where the local values of k and ε were determined from the following transport equations:

$$u \frac{\partial k}{\partial x} + v \frac{\partial k}{\partial y} = \frac{1}{y} \frac{\partial}{\partial y} \left(\frac{v_t}{\sigma_k} y \frac{\partial k}{\partial y} \right) + P - \varepsilon;$$

$$u \frac{\partial \varepsilon}{\partial x} + v \frac{\partial \varepsilon}{\partial y} = \frac{1}{y} \frac{\partial}{\partial y} \left(\frac{v_t}{\sigma_\varepsilon} y \frac{\partial \varepsilon}{\partial y} \right) + \frac{\varepsilon}{k} (c_{\varepsilon 1} P - c_{\varepsilon 2} \varepsilon),$$

and the normal stresses which appeared in equation (2) were expressed in terms of the turbulent kinetic energy as follows

$$\langle u'^2 \rangle - \langle v'^2 \rangle = c_u k.$$

Here

$$c_\mu = 0.09, \quad \sigma_k = 1, \quad \sigma_\varepsilon = 1.3,$$
$$c_{\varepsilon 1} = 1.44, \quad c_{\varepsilon 2} = 1.92.$$

Table 1 presents the calculated and experimental results for the expansion of a round jet over the main region [1–3].

The table reveals that MM significantly increases the accuracy of calculations as compared to the standard model (SM). At the same time, the analysis results showed that the values of empirical constants suggested in ref. [21]

$$c_\mu = 0.09, \quad \sigma_k = 1, \quad \sigma_\varepsilon = 1,$$
$$c_{\varepsilon 1} = 1.44, \quad c_{\varepsilon 2} = 1.90$$

Table 1. Calculated and measured forced round jet expending rates

Reference	Model	$c_{\varepsilon 2}$	$y_{0.5u}/x$
	Prediction		
	SM	1.92	0.115
[21]	MM	1.90	0.098
[23]	MSR	1.80	0.103
	Present investigations		
	SM	1.92	0.115
	MM	1.90	0.111
	MM	1.80	0.089
	Experiment		
[1]			0.086
[3]			0.087
[2]			0.089

do not provide a satisfactory agreement between the theory and experiment over the whole spectrum of characteristics. It was established that the decrease in the value of $c_{\varepsilon 2}(\sigma_\varepsilon = 1.3)$ down to 1.80, assumed in the models based on transport equations for Reynolds stresses [23], eliminates this disadvantage. However, the presence of additional parameters c_u, $c_{\varepsilon 3}$ (assumed, as in ref. [21], equal to 0.33 and 4.44, respectively) makes it impossible to come to a definite conclusion with respect to the value of $c_{\varepsilon 2}$ for the modified (k–ε) turbulence model.

Next, different levels of K_0 and E_0 were prescribed and it was established that they exert a weak influence on the dynamics of the forced jet development, with the exception of the region adjacent to the outlet, where the initial turbulence degree strongly affects the characteristics. Therefore, strictly speaking, to correctly reproduce the actual situation in mathematical models experimental distributions of K_0 and E_0 should be assigned. Downstream of the flow (for $X \geqslant 40$) the profiles of the averaged and fluctuational motion parameters disclose the similarity of the form:

$$\frac{u_c}{u_0} = \frac{6.2}{x/d_0}, \quad \frac{\langle u'v' \rangle_m}{u_0^2} = 0.015, \quad Y_{0.5u} = 0.089x, \tag{13}$$

which conforms to the results of laboratory investigations [1, 2, 12].

4.2. *Buoyant round jet*

At the second stage, calculations were performed for a vertical round jet with positive buoyancy effluxing into a stagnant medium.

Taking into account that in the process of the ASM method development, the values of empirical coefficients constantly vary, it was assumed expedient to use three groups of model constants for the analysis, namely

ASM (1)

$$\frac{1-c_0}{c_1} = 0.23, \quad c_2 = 0.51, \quad c_n = 3.0, \quad c_{n1} = 0.5,$$

$$c_k = 0.25, \quad c_\varepsilon = 0.18, \quad c_T = 0.13,$$

$$c_{T1} = 1.79, \quad c_{\varepsilon 1} = 1.44, \quad c_{\varepsilon 2} = 1.80; \tag{14a}$$

ASM (2)

$$\frac{1-c_0}{c_1} = 0.23, \quad c_2 = 0.51, \quad c_n = 2.85, \quad c_{n1} = 0.55,$$

$$c_k = 0.25, \quad c_\varepsilon = 0.15, \quad c_T = 0.11,$$

$$c_{T1} = 1.79, \quad c_{\varepsilon 1} = 1.44, \quad c_{\varepsilon 2} = 1.80; \tag{14b}$$

ASM (3)

$$\frac{1-c_0}{c_1} = 0.23, \quad c_2 = 0.51, \quad c_n = 3.2, \quad c_{n1} = 0.55,$$

$$c_k = 0.22, \quad c_\varepsilon = 0.15, \quad c_T = 0.11,$$

$$c_{T1} = 1.79, \quad c_{\varepsilon 1} = 1.44, \quad c_{\varepsilon 2} = 1.80. \tag{14c}$$

The objective of the study was to investigate the effect of free convection on the jet flow structure. It has been found that the presence of a gravitational source results in a non-monotonous dependence of $y_{0.5u}/y_{0.5\theta}$ on x_1: if in the region with the predominant effect of the forced convection $(x_1 < 0.5)(y_{0.5u}/y_{0.5\theta}) < 1$, then in the zone of the predominant effect of free convection $(x_1 > 5)(y_{0.5u}/y_{0.5\theta}) > 1$. This means that for round plumes the temperature profile is narrower than the velocity profile. The latter agrees with the results of experimental study [5, 10]. Note that within the scope of the standard ASM model (not taking into account the effect of normal Reynolds stresses on the energy redistribution in the spectrum of turbulent fluctuations) a contrary result has been obtained, i.e. in the region of a free-convective flow the velocity profile is narrower than the temperature profile even despite the introduction of corrections which hold only for the case of axial symmetry (equations (5)) [15, 16]. Next, it has been established that thermogravitational convection leads to a significant increase in the turbulent transfer level. For example, in the region of the predominance of mass forces, the Reynolds stress increases by 70%, whereas the turbulent heat flux increases by 110% as compared to similar characteristics in the region with $x_1 < 0.5$.

To estimate the reliability of the employed mathematical model, the results of numerical integration (Table 2) were compared to the experimental data (Table 3) of refs. [4–12], in which a self-similar structure of vertical round buoyant jets in uniform surroundings was investigated. The numerical data obtained indicate that at $F_0 = 5 \div 500$ the studied jet flow becomes self-similar at the distance, equal, approximately, to $x_1 \approx 5$ for mean characteristics, and to $x_1 \approx 10$ for turbulent ones. Here the behavior of numerical solutions at the jet axis can be approximated depending on x_1 by the known power functions

$$u_1 = A_u x_1^{-1/3}, \quad \theta_1 = A_\theta x_1^{-5/3}, \quad k_1 = A_k x_1^{-2/3},$$

Table 2. Asymptotic characteristics of a turbulent round buoyant jet (theoretical predictions)

Ref.	Year	Model	σ_t	$\sqrt{(\langle u'^2\rangle_c)}/u_c$	$\sqrt{(\langle v'^2\rangle_c)}/u_c$	$\sqrt{(\langle T'^2\rangle_c)}/\Delta T_c$	$(\langle u'v'\rangle_m)/u_c^2$	$(\langle v'T'\rangle_m)/u_c\Delta T_c$	A_u	A_θ	A_k	A_ε	k_c/u_c^2	$y_{0.5u}/x$	$y_{0.5\theta}/x$
[15]	1979	ASM	—	0.27	0.20	—	—	—	4.4	5.69	1.50	10.50	0.078	—	
[16]	1979	ASM	—	—	—	—	—	—	—	—	—	—	—	0.105	0.1
[17]	1983	k–L	—	—	—	—	—	—	3.8	8.1	—	—	—	0.090	0.0
[14]	1978	ASM	—	0.28	0.20	0.34	—	—	3.2	8.9	0.82	—	0.080	0.123	0.1
[18]	1984	ASM	—	—	—	—	0.014	—	4.5	12.1	1.17	—	0.058	—	
[19]	1989	ASM	—	—	0.19	—	—	—	4.25	9.9	—	—	—	0.112	0.1
[23]	1990	MSR	—	0.29	0.22	0.34	0.028	0.048	—	—	—	—	0.091	0.139	0.1
[24]	1987	MEV	—	—	—	—	—	—	3.42	9.39	1.16	9.38	0.099	—	0.1
[25]	1990	MEV	0.8	—	—	—	—	—	3.45	9.06	—	—	—	0.109	0.1
[26]	1991	MEV	0.8	—	—	—	0.023	0.033	3.48	8.79	—	—	—	0.109	0.0
[27]	1986	MEV	0.95	—	—	—	0.029	0.037	3.30	9.33	—	—	—	0.114	0.1
The		ASM(1)	—	0.26	0.20	0.40	0.029	0.041	3.18	5.85	0.78	5.77	0.077	0.131	0.1
present		ASM(2)	—	0.25	0.19	0.40	0.028	0.042	3.27	5.47	0.79	6.69	0.074	0.119	0.1
work		ASM(3)	—	0.26	0.20	0.40	0.030	0.041	3.16	5.85	0.78	5.95	0.078	0.128	0.1
[28]	1986	MEV	1.0	—	—	—	—	—	3.21	9.14	—	—	—	0.126	0.1
[13]	1982	MSR	—	0.26	0.20	—	—	—	3.50	8.22	0.91	—	0.074	0.109	0.1

$$\varepsilon_1 = A_\varepsilon x_1^{-2}, \quad u_1 = \frac{u_c}{u_0} F_D^{1/2}, \quad \theta_1 = \frac{\Delta T_c}{\Delta T_0} F_D^{1/2},$$

$$K_1 = \frac{k_c}{u_0^2} F_D, \quad \varepsilon_1 = \frac{\varepsilon_c d_0}{u_0^3} F_D^2,$$

$$x_1 = \frac{x}{d_0} F_D^{-1/2}, \quad F_D = \frac{u_0^2}{g\beta\Delta T_0 d_0}, \tag{15}$$

where the values of the found factors A_u, A_k, A_ε, A_θ are listed in Table 2. The table also includes the numerical data reported by other authors [13–19, 23–28] which were obtained with the use of turbulent models of degrees of complexity: from the simplest models of eddy viscosity (MEV), up to mathematical models including equations for turbulent stresses and flows (MSR). On the whole, one can note a fairly satisfactory correspondence between the experiments and the results of the present predictions both for mean characteristics, and for turbulent ones, although there are certain quantitative differences: the calculated values of the A_θ factor in the law of the mean excess temperature attenuation were found considerably understated. For ASM(2) at the same time, the discrepancies increase (the best coincidence of the predicted and measured results was established for the values of model constants (14c)). The same deficiency is also observed in calculations by the ASM method with the empirical corrections taken into account (5) [15, 16]. It is also worth noting that application of the model in which the equations for stresses (flows) are solved [23], only slightly improves the theoretical prediction for the given characteristic ($\theta_1 \sim 7.2x_1^{-5/3}$). Next, it should be noted that in ref. [18] the relationship $\theta_1 = 12.1x_1^{-5/3}$ was obtained numerically where the values of c_0 and c_1 were assumed equal to 0.85 and 8.2, respectively.

The relative value of the intensity of the temperature fluctuations $\sqrt{(\langle T'^2\rangle)}/\Delta T_c$, is less sensitive to the variation of model constants (14). At the flow axis it is equal to 0.40 which corresponds to the experiment [12]. At the same time, the form of the distribution of $\sqrt{(\langle T'^2\rangle)}/\Delta T_c$ differs from that which is characteristic for round plumes where $\sqrt{(\langle T'^2\rangle_c)}/\Delta T_c = \sqrt{(\langle T'^2\rangle_m)}/\Delta T_c$. The profiles of $\sqrt{(\langle T'^2\rangle)}/\Delta T_c$ for $x_1 > 5$ have the dip near the jet axis ($\sqrt{(\langle T'^2\rangle_m)}/\Delta T_c \approx 0.43$), which is however, not so large as in the region with the predominant effect of free convection ($\sqrt{(\langle T'^2\rangle_c)}/\Delta T_c = 0.15$, $\sqrt{(\langle T'^2\rangle_m)}/\Delta T_c = 0.25$).

Recently, there has been discussions in scientific literature regarding the value of the empirical constant c_{T1} or $R = 1/c_{T1}$. The authors of refs. [29–31] in their calculations employed the value $R = 0.56$; refs. [13, 15, 16] reported that jet flows with account for mass forces can be better predicted for $R = 0.8$. The experimental results of ref. [32] reveal that in jet flows with the strong effect of free convection, the value of R is much lower ($R \approx 0.25$) and it is not a universal constant. Therefore, to make the calculations of a buoyant round jet more accurate, the authors of

Table 3. Asymptotic characteristics of turbulent round buoyant jets (experimental data)

Ref.	Year	σ_t	$\surd(\langle u'^2\rangle_c)/u_c$	$\surd(\langle v'^2\rangle_c)/u_c$	$\surd(\langle T'^2\rangle_c)/\Delta T_c$	$(\langle u'v'\rangle_m)/u_c^2$	$(\langle v'T'\rangle_m)/u_c\Delta T_c$	A_u	A_θ	A_k	A_c	k_c/u_c^2	$y_{0.5u}/x$	$y_{0.5\theta}/x$
[9]	1979	0.95	0.27	0.22	0.40	0.024	0.032	—	—	—	—	0.085	—	—
[8]	1985	—	—	—	0.44	—	—	—	—	—	—	—	—	0.100
[4]	1987	—	—	—	0.40	—	—	—	9.46	—	—	—	—	0.093
[10]	1976	—	0.28	—	0.39	—	—	—	—	—	—	—	0.133	0.105
[5]	1977	—	0.28	—	0.39	—	—	3.4	9.1	—	—	—	0.112	0.103
[7]	1980	—	—	—	—	—	—	3.4	9.4	—	—	—	—	—
[6]	1977	—	—	—	—	—	—	—	7.0	—	—	—	—	0.093
[11]	1952	—	—	—	—	—	—	4.7	11.0	—	—	—	0.085	0.099
[12]	1980	—	—	—	0.39	—	—	3.5	9.35	—	—	—	0.112	0.104

refs. [14, 19] suggested considering the quantity R as a function of a certain parameter characterizing a turbulent flow. In this respect, the solution of a single equation for the dissipation rate of the quantity

$$\varepsilon_t = \frac{1}{2R}\frac{\varepsilon}{k}\langle T'^2\rangle$$

can be, apparently, a more attractive alternative [23].

At the same time, this surely, will cause new practical difficulties.

Accordingly, it is interesting to compare the above results for $\langle T'2\rangle$ with the similar value, numerically obtained within the range of the standard ASM model. Therefore, a number of additional calculations were conducted, in which the system of equations (9) was integrated, but the effect of normal stresses was not taken into account. The values of constants (4) were employed taking into account the empirical relations of the form of equation (5). It has been found that the results of numerical integration at $R = 0.8$ significantly overpredict the relative rate of fluctuations, but for $x_1 > 10$, the maximum value of $\surd(\langle T'^2\rangle)/\Delta T_c$ lies on the central line and the profiles of temperature fluctuations have the form typical for round plumes [4].

5. CONCLUSION

A modified version of an algebraic turbulence model is presented which takes into consideration the effect of normal Reynolds stresses on the energy redistribution in the spectrum of turbulent fluctuations. The modification has been carried out by accounting additional terms in the dissipation rate equation. This version of the ASM model is applied for predicting the development of turbulent round forced and buoyant vertical jets. It is shown that the model enables one to predict these jet flows with the accuracy sufficient for practical application. In particular, the model adequately accounts for the effect of free convection on the turbulent jet transfer mechanism and correctly describes the evolution dynamics of turbulent momentum and heat fluxes at the distance x, and also their variation along the coordinate y.

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